The Global Monetary System[1]

Collin Starkweather

Shanghai University / SILC
上海大学悉尼工商学院

Fall 2012

1. The International Monetary System
   - The Gold Standard
   - Bretton Woods
     - The IMF
     - The World Bank
     - The World Bank, IMF, and United Nations
   - Flexible Exchange Rates
   - Globalization and Its Discontents
2. Foreign Exchange
   - Pricing and Valuation of Foreign Exchange Products
3. Introduction to Risk Management
4. Introduction to Hedging

Introduction

We will first learn about the international monetary system, then we will discuss the implications of currency exchange rates on firms that do business in an international setting, and finally, we will discuss strategies to manage risk associated with fluctuations in exchange rates.

Definition (International Monetary System)

Institutional arrangements that govern currencies and exchange rates.

Exchange Rate Regimes

Definition (Fixed Exchange Rate)

When the values of a set of currencies are fixed against each other at some mutually agreed-on exchange rate, it it known as a *fixed exchange rate* regime.

Definition (Floating Exchange Rate)

When the foreign exchange rate for a currency is determined in the foreign exchange market, it is known as a *floating exchange rate*.

Exchange Rate Regimes (Continued)

Definition (Pegged Exchange Rate)

When the foreign exchange rate for a currency is determined as a fixed ratio relative to a reference currency, it is known as a *pegged exchange rate*.

Definition (Managed Float)

An exchange rate regime where countries try to hold the value of their currency within some range against a reference currency or "basket" of currencies is often referred to as a *managed float*. Also often referred to as a *"dirty" float.*

The Functions of Money

Economists characterize the primary functions of money as a

1. Medium of exchange
2. Store of value
3. Unit of account

The Gold Standard

The gold standard was based on a guarantee that paper currency could be converted to gold at a fixed rate of exchange; in other words, paper currency was pegged to gold [4, pp. 355-357].

- The gold standard dates back to ancient times when gold coins were used as money
- Payment for imports was made in gold or silver
- Later, as trade grew, payment was made in paper currency which was linked to gold at a fixed rate

Definition (Gold Par Value)

The amount of currency needed to purchase one ounce of gold under a gold standard.

The Gold Standard (Continued)

- In the 1880s, most trading nations used the gold standard
- The great strength of the gold standard was that it contained a powerful mechanism for achieving balance-of-trade equilibrium by all countries
  - A country experiences a balance of trade equilibrium when the income its residents earn from its exports is equal to the money its residents pay for imports; i.e., the current account of its balance of payments is in balance
- As countries monetized their war debt following World War I, the gold standard came under pressure, eventually dissolving by 1939

Trade Balances

Definition (Balance of Trade)

The difference between a country's imports and its exports. Balance of trade is the largest component of a country's balance of payments. Debit items include imports, foreign aid, domestic spending abroad and domestic investments abroad. Credit items include exports, foreign spending in the domestic economy and foreign investments in the domestic economy. A country has a trade deficit if it imports more than it exports; the opposite scenario is a trade surplus.

Definition (Balance of Payments)

A record of all transactions made between one particular country and all other countries during a specified period of time. The *balance of payments* compares the dollar difference of the amount of exports and imports, including all financial exports and imports. A negative balance of payments means that more money is flowing out of the country than coming in, and vice versa.

Bretton Woods

In 1944, representatives from 44 countries met at Bretton Woods in New Hampshire, a state on the east coast of the United States, to agree on a new international monetary system. The new agreement specified that

- Currencies were set to a fixed exchange rate
- Only the U.S. dollar was convertible to gold
- The International Monetary Fund (IMF) and the International Bank for Reconstruction and Development (IBRD), the predecessor to the World Bank, were established to maintain order in the international monetary system and promote economic development, respectively
- Countries were prohibited from using devaluations for competitive purposes and could not devalue their currencies by more than 10% without IMF approval

Note that all of the provisions of the agreements at the conference did not become fully operative until 1959.

The International Monetary Fund (IMF)

The IMF was established to maintain order in the international monetary system through the imposition of discipline on and the provision of flexibility to signatories to the Bretton Woods agreement

- *Discipline*
  - The need to maintain a fixed exchange rate put a brake on competitive devaluations and brought stability to the world trade environment
  - A fixed exchange rate regime imposed monetary discipline on countries, thereby curtailing price inflation
- *Flexibility*
  - While monetary discipline was a central objective of the agreement, a rigid policy of fixed exchange rates would be too inflexible
  - The IMF was ready to lend foreign currencies to members to tide them over during short periods of balance-of-payments deficit, when a rapid tightening of monetary or fiscal policy would hurt domestic employment

The World Bank

The World Bank was established to promote economic development. Their statement of purpose, principles, and values is as follows [2, p. 1]:

- Our dream is a world free of poverty
  - To fight poverty with passion and professionalism for lasting results
  - To help people help themselves and their environment by providing resources, sharing knowledge, building capacity, and forging partnerships in the public and private sectors
  - To be an excellent institution able to attract, excite, and nurture diverse and committed staff with exceptional skills who know how to listen and learn

The World Bank (Continued)

- Our principles
  - Client centered, working in partnership, accountable for quality results, dedicated to financial integrity and cost-effectiveness, inspired and innovative
- Our values
  - Personal honesty, integrity, commitment; working together in teams – with openness and trust; empowering others and respecting differences; encouraging risk-taking and responsibility; enjoying our work and our families

The World Bank, IMF, and United Nations

The World Bank and IMF are similar institutions in many ways

- They work closely together
- They have similar governance structures
- They have a similar relationship with the United Nations
- They have headquarters in close proximity in Washington, D.C.
- In order to be a member of the World Bank, countries must first be a member of the IMF

The World Bank, IMF, and United Nations (Continued)

The World Bank and IMF have many operational commonalities

- The IMF and World Bank Group share some common staff for administrative purposes, including the Library Network, Health Services, and the Bank/Fund Conferences Office
- The Development Committee advises the Boards of Governers of the two institutions on development issues, including trade and global environmental issues, and financial resources.
- The Boards of Governors of the World Bank Group and IMF meet annually to discuss issues related to poverty reduction and development.
- The Development Committee also meets in March or April of each year to discuss World Bank Group and IMF progress.

The World Bank, IMF, and United Nations (Continued)

Differences between the World Bank and IMF include

- The World Bank Group lends only to developing or transition economies whereas any member country may seek loans or services from the IMF
- The IMF's loans address short-term economic problems
- The IMF focuses on macroeconomic performance of the world economies and macroeconomic policy whereas the World Bank focuses on particular sectors or projects

The World Bank and the United Nations

The World Bank and United Nations have been closely tied since almost their inception at the Bretton Woods Conference

- The World Bank Group and the United Nations have almost the same membership
- A 1947 agreement recognizes the World Bank as an independent, specialized agency of the U.N.
- The World Bank Group is officially an "observer" in many U.N. bodies, including the General Assembly, currently located in New York.
- The World Bank Group has many of the same social agendas, such as poverty reduction and sustainable development, and provides country-level knowledge and policy recommendations to the U.N.
- The World Bank Group coordinates its efforts with U.N. funds for aid and implementation of various projects.

The Collapse of Bretton Woods

The Bretton Woods system collapsed on March 19, 1973 when many European countries and Japan changed their exchange rate systems from a fixed rate regime to float against the dollar and each other.

- The collapse was precipitated by an inflationary increase in the money supply and government spending by the United States to finance the Vietnam War and other policy directives
- In 1976, IMF members met in Jamaica to determine a new exchange rate system. Under the resulting Jamaica Agreement,
  - Floating rates were sanctioned
  - Gold was abandoned as a reserve asset
  - IMF annual quotas were increased to $41 billion
- Currencies have since become more volatile
- Because governments frequently intervene in the market, many people characterize the current system as a managed float system.

Fixed vs. Floating Exchange Rates

Debate concerning the relative benefits of a fixed exchange rate system (as under Bretton Woods) versus a floating rate system (as currently exists) continues.

Fixed vs. Floating Exchange Rates (Continued)

Advantages and disadvantages of a floating exchange rate system:

- Enables governments to use monetary policy to address political and economic issues
- Provides a market mechanism to reflect trade imbalances in foreign exchange rates

Fixed vs. Floating Exchange Rates (Continued)

Advantages and disadvantages of a fixed exchange rate system:

- Constrains governments' ability to use monetary policy to address political and economic issues
- Curbs volatility in exchange rates due to speculation and makes exchange rates more predictable and less uncertain
- The floating rate system is not perceived to be an effective remedy to trade imbalances

Exchange Rates in Practice

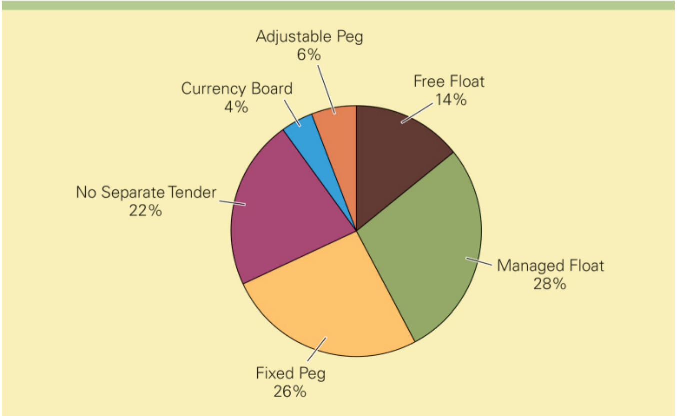

Figure: Exchange Rate Policies of IMF Members in 2006

Currency Crises

There have been a variety of stresses on the global financial system since the abandonment of the Bretton Woods system in 1973 which have contributed to foreign exchange rate volatility, including

- The partial collapse of the European Monetary System in 1992
- The Mexican currency crisis of 1995
- The Asian financial crisis of 1997
- The Russian financial crisis of 1998
- The ongoing debt crisis in Europe

Globalization and Its Discontents

The IMF has been criticized for [5, pp. 241–242]:

1. Inappropriate policies
2. Moral hazard
3. Lack of accountability

Globalization and Its Discontents (Continued)

James Wolfensohn, former President of the World Bank, felt that the current system is not managed optimally and in November 2001 advocated four improvements to the current system of governance [3, p. 31]:

1. Better policies, investment climate, and governance
2. Reduction of trade barriers
3. More development aid
4. Better international cooperation

Introduction to Foreign Exchange Markets

The foreign exchange market is composed of a variety of players who interact electronically through a variety of exchanges around the world.

Players

- Banks
- Brokers
- Dealers
- Governments
- Multinational Corporations
- Institutional Investors

Locations

- London
- Tokyo
- New York
- Singapore
- Other

Currencies Around the World

Western Currencies

- U.S.
- European
- British
- Mexican
- Brazilian
- Cuban

Eastern Currencies

- Chinese
- Japanese
- Vietnamese
- South Korean
- Australian

Currencies Around the World

Western Currencies

- U.S. Dollar (USD) $
- European Euro (EUR) €
- British Pound (GBP) £
- Mexican Peso (MXN) $
- Brazilian Real (BRL) R$
- Cuban Peso (CUP) ₱

Eastern Currencies

- Chinese 人民币 (CNY) ¥
- Japanese Yen (JPY) ¥
- Vietnamese Dong (VND) ₫
- South Korean Won (KPW) ₩
- Australian Dollar (AUD) $

Characteristics of Foreign Exchange Markets

The foreign exchange market is characterized by

- Virtually continuous trading
- Linkages that harmonize foreign exchange rates in different markets
- The majority of trades involve dollars
- Highly efficient markets in leading currencies such as dollars, euros, and yen

Characteristics of Foreign Exchange Markets

The total daily volume of the foreign exchange market in 2011 was roughly $4 trillion [1, p. 11]. As of 2007 [18, p. 262–263], roughly

- 35% of the forex market trading volume was composed of spot transactions
- 12% of the forex market trading volume was composed of forward contracts
- 53% of the forex market trading volume was composed of forex swaps

Foreign Exchange Rates

Foreign exchange markets are necessary for international trade and investment. International firms' sales, profits, and strategies are affected by the exchange rate and events in the foreign exchange market.

Definition (Foreign Exchange ("Forex") Market)

The *foreign exchange market* is a market for converting the currency of one country into the currency of another. The foreign exchange market is often referred to as the "*forex*" market.

Definition (Exchange Rate)

The *exchange rate* for a currency is the rate at which that currency is converted into another.

Foreign Exchange Rates in Practice

Exchange rates are generally quoted in European terms, except for British pounds and euros, which are typically quoted in American terms [18, p. 263]. Note that conventions are idiosyncratic and may differ between spot and forward or futures markets [11].

Definition (European Terms)

Exchange rates quoted in units of foreign currency per U.S. dollar are said to be quoted in *European terms*.

Definition (American Terms)

Exchange rates quoted in units of U.S. dollars per unit of foreign currency are said to be quoted in *American terms*.

The Foreign Exchange Market

The foreign exchange market can serve several purposes:

- Converting one currency to another
- Hedging against foreign exchange risk

Definition (Foreign Exchange Risk)

The risks associated with changes in exchange rates.

Definition (Hedge)

A *hedge* is an investment position intended to offset potential losses that may be incurred from another position or positions.

Financial Derivatives

Definition (Derivative)

A security whose price is dependent upon or derived from one or more underlying assets. The derivative itself is merely a contract between two or more parties. Its value is determined by fluctuations in the underlying asset. The most common underlying assets include stocks, bonds, commodities, currencies, interest rates and market indexes. Most derivatives are characterized by high leverage [9].

Financial Derivatives (Continued)

Derivative financial products include

- Options
- Forwards
- Futures
- Swaps

Financial Derivatives (Continued)

There are various types of options available in the financial markets, including what are commonly known as American and European options, with American options the most common exchange-traded form of option.

Definition (Option)

A financial derivative that represents a contract sold by one party to another which offers the buyer the right (but not the obligation) to buy (in the case of a *call option*) or sell (in the case of a *put option*) a security or other financial asset at an agreed-upon price (the strike price) during a certain period of time or on a specific date (exercise date) [15].

Financial Derivatives (Continued)

Definition (European Option)

The right (but not the obligation) to buy or sell an asset at a specified price on a specified expiration date.

Definition (American Option)

The right (but not the obligation) to buy or sell an asset at a specified price on or before a specified expiration date.

Financial Derivatives (Continued)

A call option is said to be "in the money" when the option's strike price is below the market price of the underlying asset and "out of the money" if it is above. Similarly, a put option is "in the money" when the option's strike price is above the market price of the underlying asset.

Definition (In the Money)

If an option is *in the money*, the sale of the option at the current market price will result in an immediate profit.

Definition (Out of the Money)

If an option would be worthless if it expired immediately, it is said to be *out of the money*.

If the strike price is equal to the market price of the asset, it is said to be "at the money." If the strike price of a call (put) option is much lower (higher) than the market price of the underlying asset, it is said to be "deep in the money."

Financial Derivatives (Continued)

Investing in an asset in the way that most people think of investing (that is, by purchasing and then holding the asset in the hope that its value will rise) is known as *going long*. A bet that is placed on the asset falling in price is known as *going short*.

Definition (Long Position)

The purchase of an asset (such as a security or currency) with the expectation that the asset will rise in value. In the context of an option, it is the purchase of an options contract.

Definition (Short Position)

The sale of a borrowed asset (such as a security or currency) with the expectation that the asset will fall in value. In the context of an option, it is the sale (also known as "writing") of an options contract.

Foreign Exchange Rates

When people discuss exchange rates, they are typically referring to spot rates. In the currency markets, exchange rates are typically quoted in European terms, except for British pounds and the euro, which are quoted in American terms.

Definition (Spot Exchange Rate)

The *spot exchange rate* is the rate at which currency can be exchanged on the market at a particular time.

Forward Exchange Rates

Forward contracts can be used to hedge against exchange rate risk.

Definition (Forward Exchange)

When two parties agree to terms by which currency will be exchanged at some specific date in the future, it is known as a forward exchange.

Definition (Forward Exchange Rate)

Exchange rates governing forward exchange transactions. Typically set in 30, 90, and 180 day increments.

Foreign Exchange Futures Markets

Futures contracts are similar to forward contracts, but are based on standardized contracts that are traded on exchanges including

- Chicago Mercantile Exchange (CME),
- Tokyo Financial Exchange,
- London International Financial Futures and Options Exchange (LIFFE), and
- IntercontinentalExchange (ICE).

Currency Futures

Futures contracts are typically marked-to-market daily and mature in quarterly intervals.

Definition (Futures Exchange)

A centralized marketplace where parties can enter into or trade standardized futures contracts or options on futures contracts.

Definition (Futures Contract)

A contract to purchase a specific amount of a commodity on a specific date at a specific price.

Forex and Currency Swaps

Foreign exchange (forex) and currency swaps are related products that are typically transacted between institutional investors, including international businesses, banks, and governments to fund foreign exchange balances and when it is desirable to move out of one currency into another for a limited period without incurring foreign exchange rate risk intervals.

Forex and Currency Swaps (Continued)

Definition (Foreign Exchange (Forex) Swap)

A forex swap is the simultaneous purchase and sale of a given amount of foreign exchange for two different value dates.

Definition (Currency Swap)

The parties to a currency swap agree to exchange the principal and/or interest of a loan in one currency for principal and/or interest of an equal net present value loan in another currency.

Managing Foreign Exchange Risk

Investors face various types of risk including

- Foreign exchange risk
  - Transaction exposure
  - Translation exposure
  - Economic exposure
- Interest rate risk
- Sovereign risk
- Counterparty and other forms of credit risk

Types of Foreign Exchange Risk

Definition (Transaction Exposure)

The extent to which profits from individual transactions are at risk to fluctuations in foreign exchange rates

Definition (Translation Exposure)

The impact of foreign exchange rate fluctuations on a firm's financial reporting

Definition (Economic Exposure)

The extent to which a firm's future international earning power is affected by changes in exchange rates

Relationship Between Interest and Exchange Rate Risk

Definition (Fisher Effect)

The Fisher Effect states that a country's nominal interest rate (i) is the sum of the real rate of interest (r) and the expected rate of inflation (I) over a given time period. Mathematically,

$$i = r + I \tag{1}$$

Definition (International Fisher Effect)

The *International Fisher Effect* states that for any two countries the spot exchange rate should change in an equal amount but in the opposite direction to the difference in nominal interest rates between two countries. Mathematically,

$$\frac{S_t - S_{t+1}}{S_{t+1}} \times 100 = i_{\$} - i_{¥} \tag{2}$$

where S_t is the spot rate at time t and S_{t+1} is the spot rate at time $t+1$ and $i_{\$}$ and $i_{¥}$ are the U.S. dollar and Japanese yen nominal interest rates over the time period from t to $t+1$.

Exchange Rate Forecasting

There are various schools of thought regarding the degree to which it is a productive exercise for firms to attempt to forecast exchange rates.

- Efficient market school
- Inefficient market school
  - Fundamental analysis
  - Technical analysis

Leading and Lagging Payments

Although difficult to implement, lead and lag strategies can mitigate foreign exchange risk.

Definition (Lead Strategy)

A lead strategy involves attempting to collect foreign currency receivables early when a foreign currency is expected to depreciate and paying foreign currency payables before they are due when a currency is expected to appreciate.

Definition (Lag Strategy)

A lag strategy involves delaying collection of foreign currency receivables if that currency is expected to appreciate and delaying payables if the currency is expected to depreciate.

Reducing Transaction and Translation Exposure

To reduce transaction and translation exposure, firms can

- Buy forward
- Use swaps
- Lead and lag payables and receivables

Reducing Economic Exposure

To reduce economic exposure, firms can

- Distribute productive assets to locations where different currencies are utilized so the firm's long-term financial well-being is not severely affected by changes in exchange rates
- Ensure assets are not too concentrated in countries where likely rises in currency values will lead to damaging increases in the foreign prices of the goods and services the firm produces

Other Strategies to Manage Foreign Exchange Risk

Generally speaking, firms should

- Strategically manage company-wide foreign exchange exposure to protect resources efficiently and ensure that various business interests and exposures are balanced
- Distinguish between transaction and translation exposure on the one hand, and economic exposure on the other hand
- Attempt to forecast future exchange rates
- Establish good reporting systems so the finance department can regularly monitor the firm's exposure position
- Produce periodic foreign exchange exposure reports

Forex Swaps and Interest Rate Parity

Forex swap spot and forward prices are generally related by *interest rate parity*, which holds that the expected return on domestic assets will equal the exchange rate-adjusted expected return on foreign currency assets.

For a forex swap of U.S. dollars for Japanese yen, interest rate parity would suggest the following relationship between forward and spot rates:

$$F = S\left(\frac{1 + i_{¥}T}{1 + i_{\$}T}\right)$$

where F is the forward rate and S is the spot rate given in yen per dollar $\left(\frac{¥}{\$}\right)$, $i_{¥}$ and $i_{\$}$ are the interest rates on Japanese yen and the U.S. dollar, respectively, and T is the time to maturity or *tenor*.

The swap points (or *pips*) are then quoted as

$$F - S = S\left(\frac{1 + i_{¥}T}{1 + i_{\$}T} - 1\right) = \frac{S(i_{¥} - i_{\$})T}{1 + i_{\$}T}$$

which is approximately $S(i_{¥} - i_{\$})T$ (i.e., proportional to the interest rate differential) when $i_{\$}T$ is small.

Example: Currency Swap

Suppose that Company A and Company B have borrowing costs in dollars and euro given in table 1.

Table: Cost of Capital Comparison

Company	**$**	**€**
Company A	5.5%	9.0%
Company B	4.0%	8.5%

Example: Currency Swap (Continued)

Company B can borrow both dollars and euros at a lower rate than Company A, so it has the *absolute advantage* in its cost of borrowing. However,

- The cost to raise dollars for Company A is 1.5% higher than for Company B; however,
- The cost to raise euros for Company A is only 0.5% higher than for Company B.
- Thus, Company A has a *comparative advantage* in borrowing euros.

Suppose Company A wants to raise dollars, whereas company B wants to raise euros. That comparative advantage can be used to both firms' mutual advantage.

- If both firms issue funds in their desired market, the total cost of raising capital would be 14.0% (= 5.5% + 8.5%); however,
- The total cost of raising capital where each has a comparative advantage is 13.0% (= 4.0% + 9.0%).
- Thus, the gain to both parties from entering a swap is 1.0% (= 14.0% - 13.0%), which can be split between the parties according to whatever terms they find mutually agreeable.
- In reality, the parties may conduct their transaction through a broker, who may also claim a portion of the gain.

If the companies agree to split the 1.0% gain evenly, the swap could be structured as follows:

- Company A could enter a swap with Company B whereby it would issue euro debt at 9.0% and agree to receive 8.0% in euro from company B in exchange for paying 4.0% in dollars.
- Company B then issues dollar debt at 4.0% and enters a swap where it promises to pay 8.0% in euro to Company A in exchange for receiving 4.0% in dollars.

In this example, the entities exchange both interest and principal, but swaps can be structured so that parties to the swap exchange only the principal, only the interest, or both the interest and principal of the loans.

Example: Currency Swap (Continued)

Swap to Company A

Operation	**€**	**$**
Issue debt	Pay €@ 9.0%	
Enter swap	Receive €@ 8.0%	Pay $@ 4.0%
Net		Pay $@ 5.0%
Direct cost		Pay $@ 5.5%
Savings		0.5%

Swap to Company B

Operation	**$**	**€**
Issue debt	Pay $@ 4.0%	
Enter swap	Receive $@ 4.0%	Pay €@ 8.0%
Net		Pay €@ 8.0%
Direct cost		Pay €@ 8.5%
Savings		0.5%

Example: Currency Swap (Continued)

If a loan for $4 million were swapped for a loan for €3 million with three annual interest payments, assuming that there is no payment of principal before the loans mature and that interest payments occur at the end of each time period, the timing of payments would be as follows:

- At time $t = 0$,
  - Company A would issue euro debt of €3 million, which means it would receive €3 million from investors in exchange for agreeing to pay them 9% (in euro) per year
  - Company A would then give the €3 million to Company B in exchange for Company B agreeing to pay it 8% (in euro) per year
  - Company B would issue dollar debt of $4 million, which means it would receive $4 million from investors in exchange for aggreeing to pay them 4% (in dollars) per year
  - Company B would likewise give the $4 million to Company A in exchange for Company A agreeing to pay it 4% (in dollars) per year
- At times $t = 1$ and $t = 2$,
  - Company A would receive €240,000 (= €3 million × 8%) in interest from Company B and pay investors interest of €270,000 (= €3 million × 9%).
  - Company A would pay $160,000 (= $4 million × 4%) in interest to Company B.
  - Company B would receive $160,000 in interest from Company A and pay investors interest of $160,000.
  - Company B would pay €240,000 in interest to Company A.
- At time $t = 3$
  - The companies would exchange their last interest payments,
  - The companies would return the principal of $4 million and €3 million to each other, and
  - The companies would return principal of $4 million and €3 million to investors.

Currency Swap Pricing

Key Concept (Equivalency of Swaps and Bond Positions)

A position in a swap receiving foreign currency is equivalent to a long position in a foreign currency bond offset by a short position in a domestic currency bond [18, p. 267].

The value V of a swap that is equivalent to a long euro bond minus a dollar bond, where $S_{\$/€}$ is the dollar price of the euro, $P_{\$}$ and $P_{€}$ are the prices of the dollar and euro bonds, is given by:

$$V = S_{\$/€} P_{€} - P_{\$}$$

Currency Swap Pricing (Continued)

If c, r, and F are the coupon, prevailing market yield, and face value of the bonds, the initial value of a swap where a company receives an interest rate of $r_{€} = 8.0\%$ in euro in exchange for paying $r_{\$} = 4.0\%$ in dollars when one dollar is worth approximately $\frac{3}{4}$ of a euro (i.e., $S_{\$/€} = \frac{3}{4}$) is given by

$$\begin{aligned} V &= \frac{4}{3} P_{€}(c_{€}, r_{€}, F_{€}) - P_{\$}(c_{\$}, r_{\$}, F_{\$}) \\ &= \frac{4}{3} P_{€}(8\%, 8\%, €75) - P_{\$}(4\%, 4\%, \$100) \\ &= \frac{\$4}{€3} €75 - \$100 = \$0 \end{aligned}$$

Currency Swap Pricing (Continued)

As can be seen from this example, which could represent the swap to Company A,

- The initial value of the swap is zero assuming a flat term structure for both countries and no credit risk
- The value of the swap can vary based on three risk factors:
  1. The spot rate $S_{\$/€}$
  2. The euro interest rate $r_{€}$
  3. The dollar interest rate $r_{\$}$

After issuance, the swap will be *in the money* for Company A if the value of the euro appreciates (i.e., if $S_{\$/€}$ increases), if the euro interest rate $r_{€}$ falls, or if the dollar interest rate $r_{\$}$ rises.

Coupon and Face Value

Definition (Coupon)

The interest rate stated on a bond at issuance. Also referred to as the "coupon rate." For example, a $1,000 bond with a coupon of 10% will pay $100 a year [8].

Definition (Face Value)

The nominal value or dollar value of a security stated by the issuer. For stocks, it is the original cost of the stock shown on the certificate. For bonds, it is the amount paid to the holder at maturity. Also known as "par value" or simply "par" [10].

Yield

Definition (Yield)

The income return on an investment. This refers to the interest or dividends received from a security and is usually expressed annually as a percentage based on the investment's cost, its current market value or its face value.

There are two stock dividend yields. If you buy a stock for $30 (cost basis) and its current price and annual dividend is $33 and $1, respectively, the "cost yield" will be 3.3% ($1/$30) and the "current yield" will be 3% ($1/$33).

Bonds have four yields: coupon (the bond interest rate fixed at issuance), current (the bond interest rate as a percentage of the current price of the bond), and yield to maturity (an estimate of what an investor will receive if the bond is held to its maturity date). Non-taxable municipal bonds will have a tax-equivalent (TE) yield determined by the investor's tax bracket [17].

Introduction to Risk Management

Definition (Risk Management)

The process by which financial risks are identified, assessed, measured, and managed in order to create economic value.

The role of the risk manager is to evaluate financial risks using both quantitative tools and judgment by [18, pp. 3,8–9]

- Identifying all risks faced by the firm
- Assessing and monitoring those risks
- Managing those risks if given the authority to do so
- Communicating these risks to the decision makers

Characterizing Risk

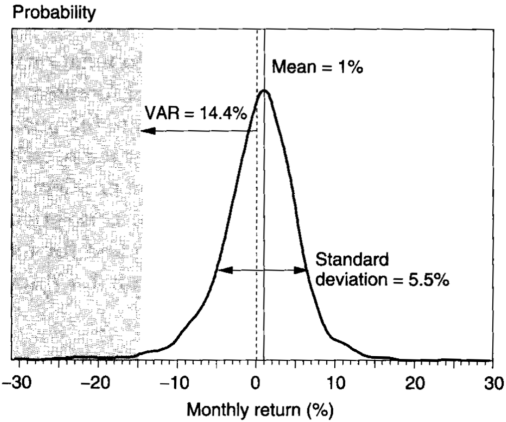

Figure: Distribution of Monthly Returns on U.S. Stocks

Characterizing Risk (Continued)

- The mean return is approximately 1% per month. This is commonly denoted by μ if we're only discussing one asset. For a portfolio of assets P, we can denote the mean return as μ_P or $\mu(R_P)$.
- The standard deviation, which is a measure of dispersion about the mean, is approximately 5.5%. This is often called volatility and is commonly denoted as σ. The portfolio variance would then be σ^2.
- The value at risk (VAR) is the cutoff point such that there is a low probability of a greater loss. Using a 99% confidence level, for example, the VAR would be roughly 14.4%.

Characterizing Risk (Continued)

Definition (Value at Risk (VAR))

A statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific time frame. Value at risk is used by risk managers in order to measure and control the level of risk which the firm undertakes. The risk manager's job is to ensure that risks are not taken beyond the level at which the firm can absorb the losses of a probable worst outcome.

VAR is measured in three variables: the amount of potential loss, the probability of that amount of loss, and the time frame [16].

Characterizing Risk (Continued)

- **Known Knowns** - Risks that are properly identified and measured, such as the distribution of stock returns.
- **Known Unknowns** - Model weaknesses that are known (or should be known) to exist but are not properly measured by risk managers. For example,
  - The risk manager could have ignored important risk factors
  - The distribution of risk factors (such as volatilities and correlations) could be measured inaccurately
  - There may be *model risk* if the mapping between positions and exposures on risk factors is incorrect
  - There may be *liquidity risk* if positions are illiquid
- **Unknown Unknowns** - Events outside the ability of risk managers to evaluate, also known as *Knightian uncertainty*. For example,
  - Changes in the regulatory environment
  - Structural changes in industry or the economy
  - Systematic counterparty risk

Characterizing Risk (Continued)

Model risk is considered a subset of operational risk, as model risk mostly affects the firm that creates and uses the model. Traders or other investors who use the model may not completely understand its assumptions and limitations, which limits the usefulness and application of the model itself.

Definition (Model Risk)

A type of risk that occurs when a financial model used to measure a firm's market risks or value transactions does not perform the tasks or capture the risks it was designed to [14].

Characterizing Risk (Continued)

Definition (Liquidity Risk)

The risk stemming from the lack of marketability of an investment that cannot be bought or sold quickly enough to prevent or minimize a loss. Liquidity risk is typically reflected in unusually wide bid-ask spreads or large price movements (especially to the downside). The rule of thumb is that the smaller the size of the security or its issuer, the larger the liquidity risk [13].

Definition (Knightian Uncertainty)

A risk that is immeasurable and cannot be accurately assessed [18, p. 8].

Absolute and Relative Risk

- Absolute risk is measured in terms of shortfall relative to the initial value of the investment. Using standard deviation as a risk measure, the absolute risk in dollar terms is

$$\sigma(\Delta P) = \sigma(\Delta P/P) \times P = \sigma(R_P) \times P$$

- Relative risk is measured relative to a benchmark index B. The deviation, also known as tracking error, is $e = R_P - R_B$, which translates to $e \times P$ in dollar terms. The relative risk, with ω representing the tracking error volatility (TEV), is

$$\sigma(e)P = [\sigma(R_P - R_B)] \times P = \omega \times P$$

Risk and Return on Two Assets

Constructing a portfolio involves combining assets with various expected returns and risk. Suppose we have two assets, a stock and a bond, with the following profile:

Table: Risk and Expected Return on Two Assets

	Average	**Volatility**	**Correlation**
Equities	11.2%	19.2%	
Long-term bonds	5.6%	8.1%	0.13

Risk and Return on Two Assets (Continued)

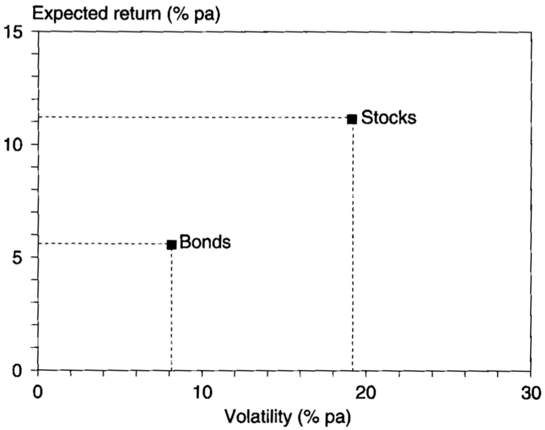

Figure: Comparing Risk and Expected Return

Risk-Adjusted Performance Measurement

- The Sharpe ratio (SR) measures the average rate of return in excess of the risk-free rate of return relative to absolute risk

$$SR = \frac{\mu(R_P) - R_F}{\sigma(R_P)}$$

- The information ratio (IR) is the ratio of the average rate of return of portfolio P in excess of the benchmark B to the TEV:

$$IR = \frac{\mu(R_P) - \mu(R_B)}{\sigma(R_P - R_B)}$$

Risk-Adjusted Performance Measurement (Continued)

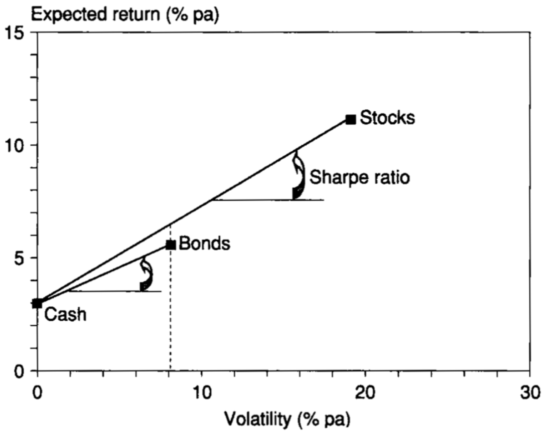

Figure: Comparing Sharpe Ratios

Constructing a Portfolio

Constructing a portfolio involves combining assets with various expected returns and risk. Suppose we have two assets, a stock and a bond, with the following profile:

Table: Absolute and Relative Performance

	Average	**Volatility**	**Performance**
Cash	3%	0%	
Portfolio P	-6%	25%	$SR =$ -0.36
Benchmark B	-10%	20%	$SR =$ -0.65
Deviation e	4%	8%	$IR = 0.50$

Variance of Correlated Random Variables

Where does the 8% come from? Recall from basic probability theory that the variance of the sum of two random variables X_1 and X_2 is given by

$$V(X_1 + X_2) = V(X_1) + V(X_2) + 2Cov(X_1, X_2)$$

where $Cov(X_1, X_2) = \rho\sqrt{V(X_1)V(X_2)}$ is the covariance of X_1 and X_2 with ρ the correlation coefficient of X_1 and X_2. Then

$$V(X_1 - X_2) = V(X_1) + V(X_2) - 2Cov(X_1, X_2)$$

Variance of Correlated Random Variables (Continued)

Assuming $\rho = 0.961$,

$$\omega^2 = \sigma_P^2 - 2\rho\sigma_P\sigma_B + \sigma_B^2$$

Since $\sigma_P = 25\%$, $\sigma_B = 20\%$, we have

$$\omega^2 = 25\%^2 - 2 \times 0.961 \times 25\% \times 20\% + 20\%^2 = 0.0064$$

Then $\omega = \sqrt{0.0064} = 0.08 = 8\%$.

Mixing Assets

Suppose you have a choice to invest in N assets. You can place weights w_i on each asset such that $\sum_{i=1}^{N} w_i = 1$. The variance of your portfolio depends on the weights as well as the correlation between the weights.

For example, if you are choosing between two investments numbered 1 and 2 (for example, 1 could be a bond and 2 a stock) the portfolio variances would be a nonlinear function of the weights

$$\sigma_P^2 = w_1^2\sigma_1^2 + 2w_1w_2(\rho\sigma_1\sigma_2) + w_2^2\sigma_2^2$$

whereas the portfolio return would be a linear function of the weights

$$\mu_P = w_1\mu_1 + w_2\mu_2$$

Mixing Assets (Continued)

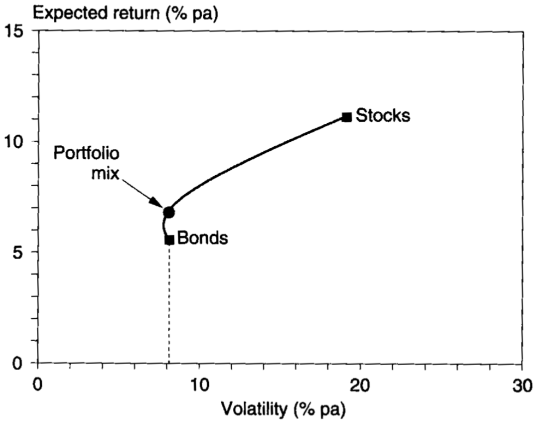

Figure: Mixing Two Assets

Example: Mixing Assets

Suppose we construct a portfolio of the equities and long-term bonds we looked at earlier which are composed of 77% bonds and 23% stocks. Then the expected return is

$$\mu_P = 0.77 \times 5.6 + 0.23 \times 11.2 = 6.9\%$$

At correlations of -1, 0.13, and 1, the portfolio variance is

$$\begin{aligned}
\rho = \quad 1 \implies \quad \sigma_P &= \sqrt{0.77^2 8.1^2 + 2 \times 0.77 \times 0.23(1.0 \times 8.1 \times 19.2) + 0.23^2 19.2^2} = 10.65 \\
\rho = \quad 0.13 \implies \quad \sigma_P &= \sqrt{0.77^2 8.1^2 + 2 \times 0.77 \times 0.23(0.13 \times 8.1 \times 19.2) + 0.23^2 19.2^2} = 8.1 \\
\rho = \quad -1 \implies \quad \sigma_P &= \sqrt{0.77^2 8.1^2 + 2 \times 0.77 \times 0.23(-1.0 \times 8.1 \times 19.2) + 0.23^2 19.2^2} = 1.8
\end{aligned}$$

Example: Mixing Assets (Continued)

Note that

- At $\rho = 0.13$, the performance of the portfolio exceeds the performance of bonds yet has the same volatility, and
- At $\rho = 0.1$, both the returns and volatility are superior to the performance of bonds.

This demonstrates the benefits of diversification.

Mean-Variance Efficient Portfolios

Consider a portfolio with a large number of assets. Assuming that all assets follow a jointly normal distribution, the distribution of portfolio returns can be summarized by only two parameters, the mean and variance.

Any portfolio that is a solution to the following optimization problem is known as a *Markowitz efficient portfolio*. The set of all such points is known as the *efficient set*.

Efficient Portfolios (Continued)

For any given return $\bar{R}$, the portfolio defined by the set of weights w that minimizes portfolio variance can be found by solving the minimization problem

$$\min_{w} \sigma_P^2$$

subject to

$$\begin{array}{lll} (1) & R_P & = \bar{R} \\ (2) & \sum_{i=1}^{N} w_i & = 1.0 \end{array}$$

Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) was developed by William Sharpe in 1964.

It postulates that stock returns are based on market returns and that the market portfolio, composed of a value-weighted average of all stocks in the portfolio, must have the highest Sharpe ratio of any feasible portfolio under the following assumptions:

1. A risk-free asset exists which can be used (at the same rate) for borrowing or lending
2. Capital markets are assumed to be perfect so that
   - There are no transaction costs
   - Securities are infinitely divisible
   - Short sales are allowed

CAPM (Continued)

Under the assumptions of the CAPM, the return $R_{i,t}$ on stock i during month t (where $t = 1 \ldots T$) relative to the return $R_{F,t}$ on the risk-free asset and market return $R_{M,t}$ is given by

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i [R_{M,t} - R_{F,t}] + \epsilon_{i,t}$$

Under this specification, β_i measures the exposure of stock i to the market factor and is known as *systematic risk*.

CAPM (Continued)

Because interactions between stock returns are due only to exposure to the market, the CAPM is a one-factor model. As a result, the covariances between any two stocks *i* and *j* are given by

$$\begin{aligned} Cov(R_i, R_j) &= Cov(\beta_i R_M + \epsilon_i, \beta_j R_M + \epsilon_j) \\ &= \beta_i \beta_j Cov(R_M, R_M) + \beta_i Cov(R_M, \epsilon_j) + \beta_j Cov(R_M, \epsilon_i) + Cov(\epsilon_i, \epsilon_j) \\ &= \beta_i \beta_j \sigma^2(R_M) \end{aligned}$$

The ϵ are uncorrelated with R_M and with each other by construction and represent *idiosyncratic risk*.

Properties of the CAPM

The Capital Asset Pricing Model gives rise to the following implications [18, p. 16]:

- The market portfolio (the value-weighted average of all stocks in the portfolio) has the highest Sharpe ratio of any feasible portfolio
  - Hence, it must be mean-variance efficient
- The *capital market line* has the highest Sharpe ratio of any portfolio on the efficient frontier.
  - Hence, it dominates any other combination of cash and stock investment
  - This demonstrates the concept of *two-fund separation*
- The mean-variance efficiency of the market implies a linear relationship between expected excess returns and the systematic risk of all stocks in the market portfolio; i.e., for any stock i, $E(R_i) = R_F + \beta_i[E(R_M) - R_F]$
  - Hence, the α_is in the CAPM model specification should theoretically all be zero
  - As a result, for actively managed portfolios, the α_is are commonly interpreted as a measure of management skill

Properties of the CAPM (Continued)

Definition (Capital Market Line)

The *capital market line* (CML) is the set of all feasible portfolios consisting of a linear combination of the risk-free asset F and the market portfolio M in the Capital Asset Pricing Model (CAPM).

Definition (Two-Fund Separation Theorem)

A theory stating that under conditions in which all investors borrow and lend at the riskless rate, all investors will either choose to possess a risk-free portfolio or the market portfolio.

Properties of the CAPM (Continued)

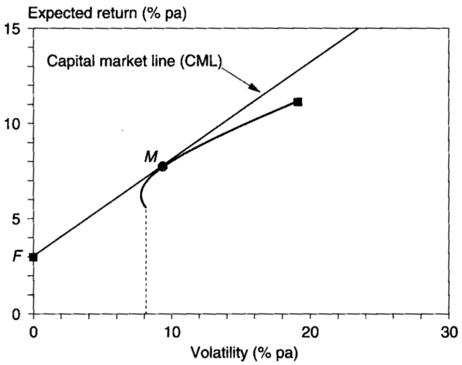

Figure: The Capital Market Line

Alternative Measures of Performance

The implications of the CAPM give rise to two alternative measures of performance:

- The Treynor ratio (TR) penalizes high β, which for well-diversified portfolios measures the contribution to the risk of the portfolio, instead of high σ, the denominator of the Sharpe ratio (SR)

$$TR = \frac{\mu(R_P) - R_F}{\beta_P}$$

- Jensen's alpha measures α, which is commonly interpreted as a measure of management skill in actively managed portfolios

$$\alpha_P = \mu(R_P) - R_F - \beta_P[\mu(R_M) - R_F]$$

Extensions of the CAPM

The CAPM has been extended by a variety of scholars since it was originally formulated, including

- Arbitrage Pricing Theory (ABT), formulated by Stephen Ross in 1976, generalizes the model to multiple factors
- Eugene Fama, famous for his efficient markets hypothesis, and Kenneth French have proposed that the CAPM should be extended to a three-factor model

Introduction to Hedging

In this section, we will discuss hedinging, which involves techniques used to determine positions that can be used to lower the risk profile of a portfolio.

First, cases where the value of the hedging instrument is linearly related to the underling risk factors, such as futures, forwards, and swaps, are examined. Then hedging using nonlinear instruments, such as options, will be reviewed.

Definition (Hedge)

Making an investment to reduce the risk of adverse price movements in an asset. Normally, a hedge consists of taking an offsetting position in a related security, such as a futures contract [12].

Example: An Exporter with Foreign Exchange Risk

Consider a U.S. exporter who has a payment of 125 million Japanese yen (¥125,000,000) that she expects to receive in seven months. The exporter decides to use an exchange-traded futures contract to hedge the receivable.

She enters into a unitary hedge by selling 10 contracts with a face amount of ¥12,500,000 on the Chicago Mercantile Exchange (CME) that expire in nine months and intends to reverse the position in seven months.

Definition (Unitary Hedge)

A hedge in which the quantity of the hedging position (i.e., the quantity underlying the offsetting position) is the same as the quantity of the position being hedged.

Example: An Exporter with Foreign Exchange Risk (Continued)

Table: Example of a Futures Hedge

Item	Initial Time	Exit Time	Gain or Loss
Market Data:			
Maturity (months)	9	2	
U.S. interest rate	6%	6%	
Yen interest rate	5%	2%	
Spot (¥/$)	¥125.00	¥150.00	
Futures (¥/$)	¥124.07	¥149.00	
Contract Data:			
Spot ($/¥)	0.008000	0.006667	-$166,667
Futures ($/¥)	0.008060	0.006711	$168,621
Basis ($/¥)	-0.000060	-0.000045	$1,954

Basis and Basis Risk

If Q is the quantity being hedged and S and F are the spot and future rates, the gain or loss on an unhedged position from time $t = 0$ to $t = T$ is

$$Q(S_T - S_0)$$

whereas with unit hedging it is

$$Q(S_T - S_0) - Q(F_T - F_0) = Q[(S_T - F_T) - (S_0 - F_0)] = Q(b_T - b_0)$$

with $b = S - F$ the *basis*.

Definition (Basis)

The variation between the spot price of a deliverable commodity and the relative price of the futures contract for the same actual that has the shortest duration until maturity [6].

Basis and Basis Risk (Continued)

Definition (Basis Risk)

The risk that offsetting investments in a hedging strategy will not experience price changes in entirely opposite directions from each other. This imperfect correlation between the two investments creates the potential for excess gains or losses in a hedging strategy, thus adding risk to the position [7].

Basis risk arises when the characteristics of the futures contract differ from those of the underlying position.

- Basis risk is lowest when the underlying position and the futures correspond to the same asset.
- Basis risk is higher with *cross-hedging*, which involves using a futures contract on a totally different asset or commodity than the cash position.

Basis and Basis Risk (Continued)

Key Concept (A Short Hedge Position is Long the Basis)

A short hedge position is long the basis; that is, it benefits when the basis widens or strengthens. This is because the position is short the hedging instrument, which falls in value (relative to the spot price) when the basis widens.

Time Components for a Hedge

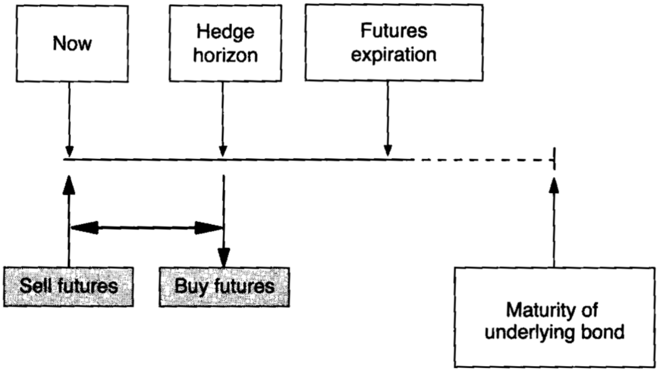

Figure: Hedging Horizon and Contract Maturity

Minimum Variance Hedging

Suppose a portfolio manager is long N futures contract such that the total change in the value of her portfolio is

$$\Delta V = \Delta S + N \Delta F$$

The portfolio manager would like to find the hedge that reduces risk to the minimum level. Variance in total profits is given by

$$\sigma^2_{\Delta V} = \sigma^2_{\Delta S} + N^2 \sigma^2_{\Delta F} + 2N\sigma_{\Delta S, \Delta F}$$

Taking the derivative with respect to N gives us

$$\frac{\partial \sigma^2_{\Delta V}}{\partial N} = 2N\sigma^2_{\Delta F} + 2\sigma_{\Delta S, \Delta F}$$

Solving for the optimal N and dropping the Δs for notational simplicity yields

$$\hat{N} = -\frac{\sigma_{\Delta S, \Delta F}}{\sigma^2_{\Delta F}} = -\frac{\sigma_{S,F}}{\sigma^2_F} = -\rho_{SF} \frac{\sigma_S}{\sigma_F}$$

Minimum Variance Hedging (Continued)

$\hat{N}$ is the *minimum variance hedge ratio*. We can rewrite the equation for $\hat{N}$ in terms of unit prices if we designate quantities as Q, unit prices as s, and the notional amount of one futures contract as f such that $S = Qs$ and $F = Q_f f$.

Then

$$\begin{aligned} \sigma_{\Delta S} &= Q\sigma(\Delta s) &&= Qs\sigma(\Delta s/s) \\ \sigma_{\Delta F} &= Q_f\sigma(\Delta f) &&= Q_f f\sigma(\Delta f/f) \\ \sigma_{\Delta S,\Delta F} & &&= \rho_{sf} Qs\sigma(\Delta s/s)Q_f f\sigma(\Delta f/f) \end{aligned}$$

and if β_{sf} is the coefficient in the regression of $\Delta s/s$ on $\Delta f/f$,

$$\hat{N} = -\rho_{SF}\frac{Qs\sigma(\Delta s/s)}{Q_f f\sigma(\Delta f/f)} = -\rho_{SF}\frac{\sigma(\Delta s/s)}{\sigma(\Delta f/f)}\frac{Qs}{Q_f f} = -\beta_{sf}\frac{Q\times s}{Q_f\times f}$$

Minimum Variance Hedging (Continued)

Definition (Effectiveness of a Hedge)

The proportional reduction in variance due to the hedging of a position is often called the *effectiveness* of the hedge.

Key Concept (Best Hedge)

The optimal hedge (in terms of variance minimization) is given by the negative of the beta coefficient of a regression of changes in the cash value on changes in the payoff on the hedging instrument.

Beta Hedging

Suppose that you want to hedge stocks. Beta reflects the exposure to the rate of return on a portfolio of stocks i to movements in the market m:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

where β represents systematic risk, α can be ignored since it is not a source of risk, and ϵ is uncorrelated to market movements.

Then

$$(\Delta S/S) \approx \beta \times (\Delta M/M)$$

Beta Hedging (Continued)

Now assume we have a stock index futures contract with a beta of unity so that

$$(\Delta F/F) = 1.0 \times (\Delta M/M)$$

Then the total portfolio payoff is

$$\begin{aligned}\Delta V &= \Delta S + N\Delta F \\ &= \beta S(\Delta M/M) + NF(\Delta M/M) \\ &= (\beta S + NF) \times (\Delta M/M)\end{aligned}$$

Beta Hedging (Continued)

When $(\beta S + NF)$ is zero, then the net exposure is zero. So the optimal number of contracts to short is given by

$$\hat{N} = -\frac{\beta S}{F}$$

Key Concept (Beta Hedging)

The optimal hedge with stock index futures is given by the beta of the cash position times its value divided by the notional of the futures contract.

Nonlinear (Option) Risk Models

Suppose the value of an option whose underlying asset is a foreign currency can be written as the function

$$f_t = f(S_t, r_t, r_t^*, \sigma_t, K, \tau)$$

where

$$\begin{aligned}
S_t &= \text{current spot price of the assets in dollars} \\
K &= \text{exercise price of the option contract} \\
f_t &= \text{current value of the derivative instrument} \\
r_t &= \text{domestic risk-free interest rate} \\
r_t^* &= \text{foreign risk-free interest rate (also denoted } y\text{)} \\
\sigma_t &= \text{annual volatility of the rate of change in } S \\
\tau &= \text{time to maturity}
\end{aligned}$$

Taylor Expansion

The change in the value of the financial derivative based on *local* movements in the arguments to the function can be described using a Taylor expansion

$$df = \frac{\partial f}{\partial S} dS + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} dS^2 + \frac{\partial f}{\partial r} dr + \frac{\partial f}{\partial r^*} dr^* + \frac{\partial f}{\partial \sigma} d\sigma + \frac{\partial f}{\partial \tau} d\tau + \dots$$

The Taylor expansion is not a good approximation if

1. There are large movements in the underlying risk factor
2. There are highly nonlinear exposures, such as options near expiration or exotic options
3. There are strong cross-partial effects, such as σ changing in a correlated fashion with S

Option Pricing

The price of a European stock option can be modeled with the Black-Scholes equation, which is based on the following assumptions [18, pp. 192–193]

1. The price of the underlying asset moves in a continuous fashion
2. Interest rates are known and constant
3. The variance of underlying asset returns is constant
4. Capital markets are perfect
5. Asset prices evolve by a process that can be modeled by geometric Brownian motion

Option Pricing (Continued)

Geometric Brownian motion describes a return generating processes that has the following properties:

- The total return can be described by the differential equation

$$\frac{dS}{S} = \mu dt + \sigma dz$$

  where μdt represents the drift and dz represents a stochastic component with mean zero and variance dt.
- Thus, over a short time interval dt, the price can be represented by a random variable whose logarithm is normally distributed with mean μdt and variance σdt.

Option Pricing (Continued)

Based on these assumptions, the closed-form solution for the value of a European call option on a non-dividend paying stock is

$$c = SN(d_1) - Ke^{-r\tau}N(d_2)$$

where S is the spot price of the stock, K is the strike price of the option, r is the risk-free interest rate, τ is the time until expiration, and $N(d)$ is the cumulative distribution function for the standard normal distribution, such that

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{1}{2}x^2} dx$$

Option Pricing (Continued)

The values of d_1 and d_2 are given by

$$d_1 = \frac{ln(S/Ke^{-r\tau})}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

The closed-form solution for the value of a European put option on a non-dividend paying stock is

$$p = Ke^{-r\tau}N(-d_2) - SN(-d_1)$$

Option Greeks: Delta and Gamma

Changes in the prices of options with respect to changes in the price of the underlying are commonly denoted by greek letters and are hence colloquially referred to as “the greeks.”

Based on the Black-Scholes equation, the first partial derivative of a call option with respect to the price of the underlying, called the *delta*, is given by

$$\Delta_c = \frac{\partial c}{\partial S} = e^{-r^*\tau} N(d_1)$$

The delta of a put option is given by

$$\Delta_p = \frac{\partial p}{\partial S} = e^{-r^*\tau} [N(d_1) - 1]$$

Option Greeks: Delta and Gamma (Continued)

The gamma of a call or put option is the second-order derivative with respect to the price of the underlying and is given by

$$\Gamma = \frac{\partial^2 c}{\partial S^2} = \frac{\partial^2 p}{\partial S^2} = \frac{e^{-r^*\tau}\Phi(d_1)}{S\sigma\sqrt{\tau}}$$

with Φ the normal density function.

Option Greeks: Delta and Gamma (Continued)

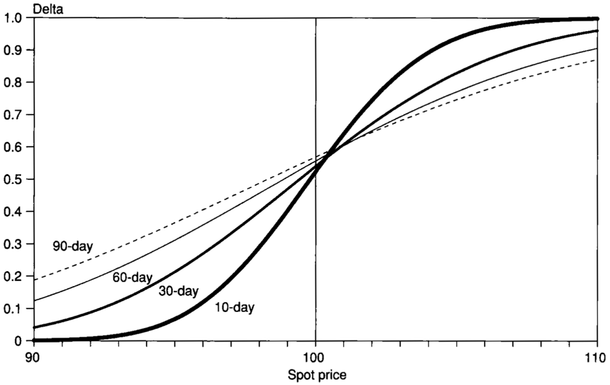

Figure: Option Delta

Option Greeks: Delta and Gamma (Continued)

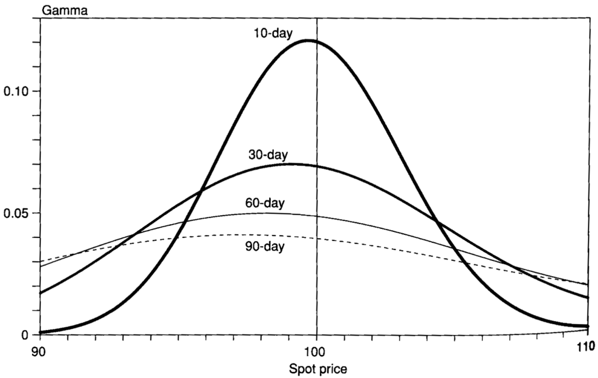

Figure: Option Gamma

Delta-Gamma Approximation for a Long Call

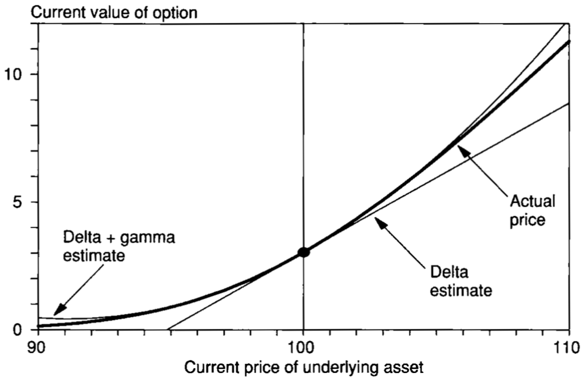

Figure: Delta-Gamma Approximation for a Long Call

Option Greeks: Delta and Gamma (Continued)

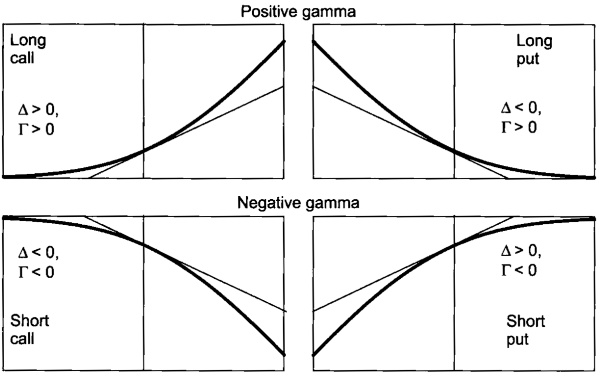

Figure: Delta and Gamma of Option Positions

Option Greeks: Delta and Gamma (Continued)

Key Concept (Delta)

The delta of an at-the-money call option is close to $\frac{1}{2}$. *Delta moves to 1 as the call goes deep in-the-money. It moves to zero as the call goes deep out-of-the-money.*

Similarly, the delta of an at-the-money put option is close to $-\frac{1}{2}$. *Delta moves to -1 as the put goes deep in-the-money. It moves to zero as the put goes deep out-of-the-money.*

Key Concept (Gamma)

For vanilla options, gamma is the highest, or nonlinearities are most pronounced, for short-term at-the-money options.

Delta Hedging

The Black-Scholes equation is derived from a set of differential equations based on an assumed delta neutral hedge that produces arbitrage free option pricing.

Definition (Delta Neutral Hedge)

A delta neutral hedge is one in which the value of the hedge moves in tandem with the value of the underlying asset so that the delta of the portfolio is as close to zero as possible.

The positive and negative deltas of a delta neutral portfolio would balance to result in a net change to the overall position of zero.

Delta Hedging (Continued)

If c_i is the price of a call option on stock i at price S_i, and if the Taylor expansion of the change in the option price is given by

$$c_i(S_i + \epsilon) = c_i(S_i) + \epsilon \frac{dc_i}{dS_i} + \frac{1}{2}\epsilon^2 \frac{d^2 c_i}{dS_i^2} + \dots \qquad (3)$$

then the term $\frac{dc_i}{dS_i}$ is referred to as the Δ, or "delta."

Recall that the delta of a call option would be expected to be positive while the delta of a put option would be expected to be negative. Everything else constant, purchasing $\frac{-1}{\Delta}$ put options to hedge a long position in i would produce a delta neutral hedge for small movements in the market price of the shares of i over a short period of time.

Option Greeks: Vega

Recall that option value is sensitive to volatility as well as the price of the underlying, interest rates, and the passage of time.

Definition (Vega)

The sensitivity of an option with respect to volatility is called *vega* and is commonly denoted by the greek letter "nu" (ν). Vega is sometimes also called lambda (Λ) or kappa (κ).

As with gamma (Γ), vega has the bell shape of the normal density function Φ.

Option Greeks: Vega (Continued)

The vega of a European call or put is given by

$$\nu = \frac{\partial c}{\partial \sigma} = \frac{\partial p}{\partial \sigma} = S e^{-r^* \tau} \sqrt{\tau} \Phi(d_1)$$

Key Concept (Vega)

Vega is highest for long-term at-the-money options.

Option Greeks: Vega (Continued)

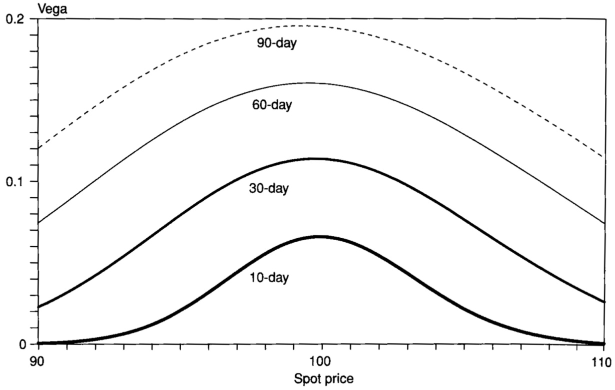

Figure: Option Vega

Option Greeks: Rho

Recall that option value is sensitive to interest rates as well as the price of the underlying, volatility, and the passage of time.

Definition (Rho)

The sensitivity of an option with respect to interest rates is called *rho* (ρ).

Option Greeks: Rho (Continued)

The rho of a European call on an asset denominated in a foreign currency with respect to the domestic interest rate is given by

$$\rho_c = \frac{\partial c}{\partial r} = Ke^{-r\tau}\tau N(d_2)$$

The rho of a European put on an asset denominated in a foreign currency with respect to the domestic interest rate is given by

$$\rho_p = \frac{\partial p}{\partial r} = Ke^{-r\tau}\tau N(-d_2)$$

Option Greeks: Rho (Continued)

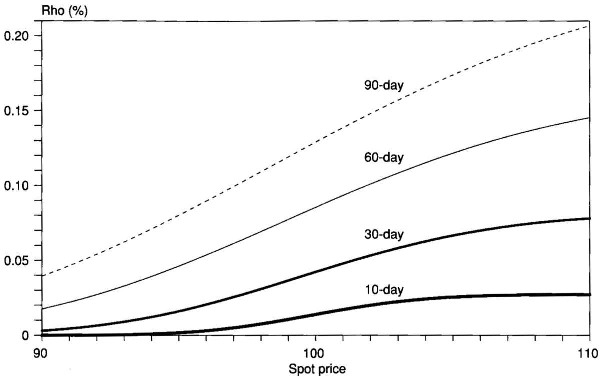

Figure: Call Option Rho with Respect to the Domestic Interest Rate

Option Greeks: Rho (Continued)

The rho of a European call with respect to the foreign interest rate (sometimes called phi, or ϕ) is given by

$$\rho_c^* = \frac{\partial c}{\partial r^*} = -Se^{-r^*\tau}\tau N(d_1)$$

The rho of a European put with respect to the foreign interest rate (sometimes called phi, or ϕ) is given by

$$\rho_p^* = \frac{\partial p}{\partial r^*} = Se^{-r^*\tau}\tau N(-d_1)$$

Option Greeks: Rho (Continued)

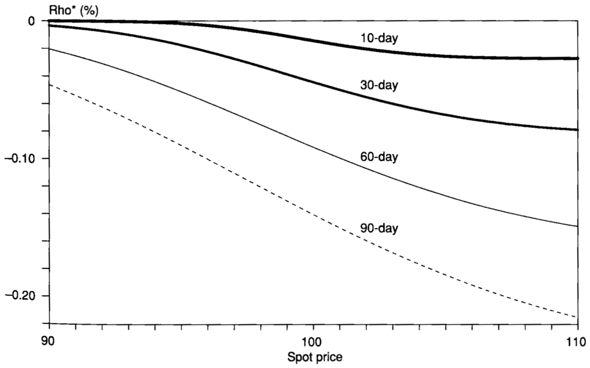

Figure: Call Option Rho with Respect to the Foreign Interest Rate

Option Greeks: Theta

Recall that option value is sensitive to the passage of time as well as the price of the underlying, volatility, and interest rates.

Definition (Theta)

The variation in option value due to the passage of time is called *theta*. It is also known as the *time decay*.

Note that time is not a risk factor as the passage of time is perfectly predictable.

Option Greeks: Theta (Continued)

For a European call, theta is given by

$$\Theta_c = \frac{\partial c}{\partial \tau} = -\frac{Se^{-r^*\tau}\sigma\Phi(d_1)}{2\sqrt{\tau}} + r^*Se^{-r^*\tau}N(d_1) - rKe^{-r\tau}N(d_2)$$

For a European put, theta is given by

$$\Theta_p = \frac{\partial p}{\partial \tau} = -\frac{Se^{-r^*\tau}\sigma\Phi(d_1)}{2\sqrt{\tau}} - r^*Se^{-r^*\tau}N(-d_1) + rKe^{-r\tau}N(-d_2)$$

Option Greeks: Theta (Continued)

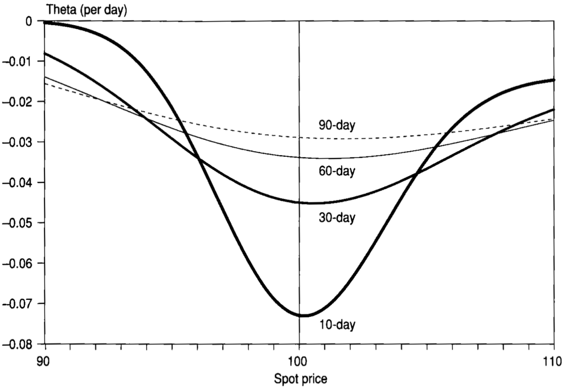

Figure: Option Theta

Delta Hedging and the Option Greeks

The Black-Scholes equation is the solution to a partial differential equation (PDE) known as the Black-Scholes partial differential equation given by

$$(r - y)\Delta S + \frac{1}{2}\Gamma\sigma^2 S^2 + \Theta = rf$$

This PDE can be derived from the equation for the price evolution of an asset by geometric Brownian motion by creating a hypothetical asset (the option) that produces a risk-free portfolio and returns the risk-free rate of return.

Delta Hedging and the Option Greeks (Continued)

Consider a portfolio of derivatives on the same underlying asset that is delta hedged. Then with $\Delta = 0$,

$$\frac{1}{2}\Gamma\sigma^2 S^2 + \Theta = rf$$

Thus, when Γ is large and positive, Θ must be negative if rf is small, and a delta-hedged position with positive gamma must have negative time decay (Θ).

Key Concept (Γ and Δ in Delta-Hedged Portfolios)

For delta-hedged portfolios, Γ and Θ must have opposite signs. Portfolios with positive gamma must experience time decay.

References I

Marc Auboin.
Use of currencies in international trade: Any changes in the picture?
Staff Working Paper ERSD-2012-10, Economic Research and Statitics Division, World Trade Organization, May 2012.

World Bank.
A Guide to the World Bank.
The World Bank, Washington, D.C., 2003.

Peter Van den Bossche.
The Law and Policy of the World Trade Organization.
Cambridge University Press, Cambridge, England, 2005.

References II

Charles W. L. Hill.
International Business: Competing in the Global Marketplace.
McGraw-Hill Companies, Inc., New York, New York, 7th edition, 2009.

Charles W. L. Hill.
International Business: Competing in the Global Marketplace.
McGraw-Hill Companies, Inc., Beijing, China, 7th edition, 2009.

Investopedia.
Basis.
December 6, 2012.

References III

Investopedia.
Basis risk.
December 6, 2012.

Investopedia.
Coupon.
December 14, 2012.

Investopedia.
Derivative.
December 12, 2012.

Investopedia.
Face value.
December 14, 2012.

References IV

Investopedia.
Forex tutorial: Reading a forex quote and understanding the jargon.
December 12, 2012.

Investopedia.
Hedge.
December 6, 2012.

Investopedia.
Liquidity risk.
December 4, 2012.

References V

Investopedia.
Model risk.
December 4, 2012.

Investopedia.
Option.
December 12, 2012.

Investopedia.
Value at risk.
December 6, 2012.

Investopedia.
Yield.
December 14, 2012.

References VI

Philippe Jorion.
Financial Risk Manager Handbook.
John Wiley & Sons, Inc., Hoboken, New Jersey, 6th edition, 2011.

The Global Monetary System[2]

Collin Starkweather

Shanghai University / SILC
上海大学悉尼工商学院

Fall 2012